Set A (7) 2019

Sr. No. of Question Paper : 3658

Unique Paper Code : 235204

Name of the Paper : Probability & Statistics-MAHT 203

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II

Duration : 3 Hours

Maximum Marks: 75

Instructions for Candidates

a) Write your Roll No. on the top immediately on receipt of this question paper.
b) In all there are **six** questions.
c) Question No. 1 is compulsory and it contains five parts of **3** marks each.
d) In Question No. **2** to **6**, attempt any **two** parts from **three** parts. Each part carries **6** marks.
e) Use of scientific calculator is allowed.

1. (i) If C_1 and C_2 are events in a sample space S. Then prove that

$$P(C_1 \cap C_2) \geq P(C_1) + P(C_2) - 1.$$

(ii) A bowl contains 16 chips, of which 6 are red, 7 are white and 3 are blue. If 4 chips are taken at random and without replacement, find the probability that : (a) each of the 4 chips is red; (b) None of 4 chips is red.

(iii) Let X be a random variable with cdf F_X.

Then for a < b . $P\ (a < X \leq b) = F_X(b) - F_X(a)$.

(iv) Let X has a negative exponential distribution with parameterλ. If $P(X \leq 1) = P(X > 1)$, what is the variance of X.

(v) Show that if a random variable has a uniform density with the parameters α and β, the probability that it will take on a value less than $\alpha + p(\beta - \alpha)$ is equal to p.

2. (a) State and prove Boole's inequality.

(b) Cast a dice a number of independent times till a six appears on the up face of the dice

(i) Find the pmf p(x) of X, the number of casts needed to obtain first six

(ii) Show that $\sum_{x=1}^{\infty} p(x) = 1$

(c)) Let a random variable X has pmf given by $p(x) = \begin{cases} \frac{1}{3} & ; x = -1,0,1 \\ 0 ; & otherwise \end{cases}$

Find the cdf F(x) of X.

3. (a) Let the pmf $p(x)$ be positive at $x = -1,0,1$ and zero elsewhere

(i) If $p(0) = \frac{1}{4}$. Find $E(X^2)$

(ii)) If $p(0) = \frac{1}{4}$ and $E(X) = \frac{1}{4}$. Determine p(-1) and p(1)

(b) Prove that moment generating function of Poisson distribution is given by $M_X(t) = e^{\lambda(e^t - 1)}$. Hence find its mean and variance.

(c) Let X have pdf $f(x) = 3x^2 : 0 < x < 1$, zero elsewhere. Consider a random rectangle whose sides are X and (1-X). Determine the expected value of the area of the rectangle.

4. (a) Let X has the pdf given by $f(x) = \begin{cases} cx^3 ; & 0 < x < 2 \\ 0 & otherwise \end{cases}$

Find the

(i) Constant c

(ii) $P(\frac{1}{4} < X < 1)$

(b) Prove that the mean and variance of the Uniform distribution are given by

$$\mu = \frac{\alpha+\beta}{2}, \quad \sigma^2 = \frac{(\beta-\alpha)^2}{12}$$

(c) Prove that the moment generating function of Normal distribution is given by

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

5.(a) The joint pdf of random variables X and Y is given by
$$f(x,y) = \begin{cases} x+y; & 0<x<1, 0<y<1 \\ 0; & otherwise \end{cases}$$

Find the conditional mean and variance of Y, given X= x, $0 < x < 1$.

(b) Define the independence of two variables X_1 and X_2. Suppose X_1, X_2 have the joint cdf $F(x_1, x_2)$ and marginal cdfs $F_1(x_1)$ and $F_2(x_2)$ respectively. Show that the variables X_1 and X_2 are independent if and only if $F(x_1, x_2) = F_1(x_1)\,F_2(x_2)$ for all $(x_1, x_2) \in R^2$.

(c) Suppose the random variables X and Y have the joint density given by

$$f(x,y) = \begin{cases} xe^{-x(1+y)}; & x>0,\ y>0 \\ 0 & otherwise \end{cases}$$. Find the regression equation $\mu_{Y|X}$ of Y on X.

Also sketch the regression curve.

6. (a) State and Prove Chapman-Kolmogorov's equations.

(b) Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500 :

(i) What can be said about the probability that this week's production will be atleast 1000?

(ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600?

(c) State and prove central limit theorem for independent, identically distributed random variables with finite variance.